Natural little hierarchy from a partially goldstone twin Higgs

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ABSTRACT: We construct a simple theory in which the fine-tuning of the standard model is significantly reduced. Radiative corrections to the quadratic part of the scalar potential are constrained to be symmetric under a global $U(4) \times U(4)'$ symmetry due to a discrete Z_2 "twin" parity, while the quartic part does not possess this symmetry. As a consequence, when the global symmetry is broken the Higgs fields emerge as light pseudo-Goldstone bosons, but with sizable quartic self-interactions. This structure allows the cutoff scale, Λ, to be raised to the multi-TeV region without significant fine-tuning. In the minimal version of the theory, the amount of fine-tuning is about 15% for $\Lambda = 5\,\text{TeV}$, while it is about 30% in an extended model. This provides a solution to the little hierarchy problem. In the minimal model, the "visible" particle content is exactly that of the two Higgs doublet standard model, while the extended model also contains extra vector-like fermions with masses $\approx (1 \sim 2)\,\text{TeV}$. At the LHC, our minimal model may appear exactly as the two Higgs doublet standard model, and new physics responsible for cutting off the divergences of the Higgs mass-squared parameter may not be discovered. Several possible processes that may be used to discriminate our model from the simple two Higgs doublet model are discussed for the LHC and for a linear collider.

KEYWORDS: Beyond Standard Model, Higgs Physics.

Contents

1. Introduction

Despite its tremendous phenomenological success, the standard model is an incomplete theory. In the standard model, the Higgs mass-squared parameter receives radiative corrections of order the cutoff scale squared, implying the existence of some new physics at a scale not much larger than the scale of electroweak symmetry breaking. On the other hand, experiments have not found any convincing sign of such physics so far: the scale suppressing nonrenormalizable operators must be larger than several TeV. This suggests that the new physics must cut off the corrections to the Higgs mass-squared parameter without much affecting the other sectors of the standard model. What is this new physics and how can we find it?

An interesting idea to control radiative corrections to the Higgs potential is to consider it to be the pseudo-Goldstone boson (PGB) of some broken global symmetry [1]. The actual implementation of this idea, however, is not so simple. The Higgs potential possesses a global symmetry at tree level, which is explicitly broken by the electroweak gauge and Yukawa interactions. These explicit breakings then generate the potential for the Higgs field h at loop level. This itself, however, does not help much because the generated Higgs mass-squared parameter m_h^2 is of order $L\Lambda^2$, where Λ is the cutoff and L is the one-loop factor: $L \approx g^2 C/16\pi^2$ or $y^2 N/16\pi^2$ with g and y gauge and Yukawa couplings and C and N multiplicity factors. We have just dropped the tree-level m_h^2 term in the standard model simply by declaring that the Higgs is a PGB. Some progress, however, can be made if we control radiative corrections from gauge and Yukawa interactions either by breaking symmetry collectively [2], by making the size of the gauge group generating the PGB Higgs large and thus separating the momentum cutoff scale from the cutoff of the theory [3], or

by using a discrete symmetry [4]. In these cases, the correction to m_h^2 can be cut off below the real cutoff of the theory Λ, so that we can have a perturbative theory describing physics above the naive one-loop cutoff scale of the standard model. The question for the consistency with the data can then be addressed by studying this perturbative physics.

A generic problem for these classes of theories is that since the generated Higgs potential is a function of $\cos(h/f)$ and $\sin(h/f)$, where f is the decay constant for the PGB Higgs, the vacuum expectation value (VEV) of h is naively of order f: $\langle h \rangle \approx f$. This is not good because the cutoff scale Λ is then, at most, of order $4\pi f \approx 2$ TeV, so that it does not help to understand why the deviations from the standard model are experimentally so small. There are essentially two ways to evade this problem. One is to invoke a cancellation in the quadratic term in the Higgs potential. The one-loop potential for the PGB Higgs is schematically written as $V(h) = L(-\eta_2 f^2 |h|^2 + \eta_4 |h|^4/2 - \eta_6 |h|^6/6f^2 + \cdots)$, where η's are naturally of $O(1)$. Then, if the coefficient η_2 is somehow small, for example due to a cancellation between gauge and Yukawa contributions, or if we add an additional term of the form $\delta V(h) = \mu^2 |h|^2$ such that $\mu^2 \approx \eta_2 f^2 L$, we can obtain $\langle h \rangle \ll f$ and push up the cutoff scale Λ to be larger than 2 TeV. The other possibility is to introduce an extra quartic coupling, $\delta V(h) = \lambda |h|^4/2$. This would be interesting because we may then obtain $\langle h \rangle \approx f L^{1/2} \ll f$ for $\lambda = O(1)$, so that the cutoff may be pushed up to $\Lambda \approx \langle h \rangle/L \gg 2$ TeV without unnatural cancellations. The problem is that such a quartic coupling also gives a correction to the Higgs mass-squared parameter. If we set the cutoff to be $\Lambda \approx 4\pi f$, the correction is of order $\delta m_h^2 \approx (\lambda/16\pi^2)\Lambda^2 \approx \lambda^2 f^2$, which is much larger than the corresponding term in the original potential $V(h)$. In order to make this possibility work, therefore, we need some mechanism controlling this correction. One such mechanism is collective symmetry breaking in little Higgs theories. Implementing it in realistic theories, however, generically requires some non-trivial model building efforts [2, 5]. Moreover, the constraints from the precision electroweak data are often quite severe [6], requiring a further ingredient, such as T parity [7], to make the models fully viable.

In this paper, we construct a theory which addresses the issues described above. An important ingredient for this construction is the discrete Z_2 "twin" symmetry relating the standard-model fields with their mirror partners. It has recently been shown in [4] that this symmetry can be used to control divergences from the gauge and Yukawa couplings in PGB Higgs theories. Using this "twin Higgs" mechanism, we can construct a simple theory which naturally realizes electroweak symmetry breaking. We show that by introducing an operator that explicitly violates the global symmetry but still preserves the Z_2 symmetry, we can generate an order-one quartic coupling in the Higgs potential without giving a quadratically divergent contribution to the Higgs mass-squared parameter. This allows us to push up the cutoff scale to the multi-TeV region without significant fine-tuning, and thus to solve the little hierarchy problem implied by the mismatch between the stability of the electroweak scale and the constraints from experiments [8]. With an extended top quark sector and a mild tuning of order 10%, this basic framework allows the cutoff scale as high as about 8 TeV. We assume that our theory is weakly coupled at the cutoff scale, although it may be possible to extend it to the strongly coupled case. An interesting aspect of the model is that the scalar potential does not possess any approximate continuous global

symmetry. The global symmetry is explicitly broken by an $O(1)$ amount by a dimensionless quartic coupling. The gauge and Yukawa interactions also break the symmetry by an $O(1)$ amount. Nevertheless, the quadratic terms in the scalar potential possess an enhanced global symmetry, guaranteed by the discrete Z_2 "twin" symmetry, and this partial global symmetry is sufficient to achieve our goals. The theory has two Higgs doublets, whose couplings to matter fields can take either a Type-I, Type-II or mixed form.

The minimal version of our theory may lead to a potentially embarrassing situation at the LHC. While the theory does not have significant fine-tuning in electroweak symmetry breaking, the LHC may just see the (two Higgs doublet) standard model, and may not find any new physics responsible for cutting off the divergences of the Higgs mass-squared parameter. This is because divergences in the Higgs mass-squared parameter due to the standard model fields are canceled by fields that are singlet under the standard model gauge group. The deviations from the simple two Higgs doublet model due to these singlet fields can be very small at the LHC. The deviations, however, may show up at a linear collider. This demonstrates that it may be too early to give up the concept of naturalness even if the LHC does not find any new physics associated with the cancellation of the Higgs mass divergences. Precision Higgs studies at a linear collider may be necessary before any firm conclusion can be drawn.

In a version of the theory in which the top quark sector is extended and the amount of fine-tuning is further reduced, we will find new strongly interacting vector-like fermions at the LHC, which are responsible for cutting off the radiative correction to the Higgs mass-squared parameter from the top quark. These particles, however, may be the only new particle we will find at the LHC beyond the two Higgs doublets, because all the other divergences in the standard model can be canceled by fields which are singlet under the standard model gauge group.

The organization of the paper is as follows. In the next section we describe the basic structure of our theory. In section 3 we calculate radiative corrections to the Higgs potential and show that the cutoff scale can be pushed up to the multi-TeV region without significant fine-tuning. In section 4 we extend our minimal model so that it allows a smaller fine-tuning and/or larger cutoff scale. We find that the cutoff scale can be raised up to about 8 TeV with a mild tuning of order 10%. In section 5 we discuss the possibility of making the theory strongly coupled at Λ. Phenomenology of the model is discussed in section 6, and conclusions are given in section 7.

2. Minimal theory

We consider that our theory is an effective field theory describing physics below the cutoff scale Λ, which is given by specifying the lagrangian at the scale Λ. We assume that the theory is weakly coupled at Λ, and that radiative corrections to the Higgs mass-squared parameter (at least power divergent ones) are cut off at this scale. We do not need to specify physics above Λ for the present purpose. As we will see later, the scale Λ in our theory can be in the multi-TeV region without significant fine-tuning.

Let us consider two scalar fields Φ and Φ' that transform as fundamental four-dimensional representations under global $U(4)$ and $U(4)'$ symmetries, respectively. We assume that the tree-level potential for Φ and Φ' drive non-zero VEVs for Φ and Φ', breaking $U(4) \to U(3)$ and $U(4)' \to U(3)'$, respectively. The $U(4) \times U(4)'$ invariant lagrangian causing such a breaking pattern is[1]

$$\mathcal{L} = -\eta(|\Phi|^2 - f^2)^2 - \eta'(|\Phi'|^2 - f'^2)^2. \tag{2.1}$$

What are the sizes for η, η', f and f'? We take $\eta \sim \eta' = O(1)$ because the theory is assumed to be weakly coupled at the scale Λ. For f and f', we take them to be somewhat smaller than the cutoff scale Λ: $f \sim f' = O(\Lambda/4\pi)$. This is crucial to achieve our goal of raising the cutoff, as will become clear later. These values of f and f' are stable under radiative corrections, i.e. technically natural. They may naturally arise if Φ and Φ' themselves are PGBs of some larger global group, say those of $U(5) \times U(5)' \to U(4) \times U(4)'$, but here we simply take $f \sim f' \sim \Lambda/4\pi$ without specifying their origin.

We denote the upper and lower halves of the Φ (Φ') field as H_A and H_B (H_A' and H_B'), respectively. When Φ and Φ' develop VEVs

$$\langle\Phi\rangle = \left\langle \begin{pmatrix} H_A \\ \hline H_B \end{pmatrix} \right\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \qquad \langle\Phi'\rangle = \left\langle \begin{pmatrix} H_A' \\ \hline H_B' \end{pmatrix} \right\rangle = \begin{pmatrix} 0 \\ 0 \\ f' \\ 0 \end{pmatrix}, \tag{2.2}$$

14 Goldstone bosons appear associated with the breaking $U(4) \times U(4)' \to U(3) \times U(3)'$. Now, we gauge the $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$ subgroup of $U(4) \times U(4)'$. Here, $SU(2)_A \times U(1)_A$ acts on the upper half components of Φ and Φ' such that both H_A and H_A' have the quantum numbers of $\mathbf{2}_{-1/2}$, while $SU(2)_B \times U(1)_B$ on the lower half components of Φ and Φ' such that H_B and H_B' transform as $\mathbf{2}_{-1/2}$. This gauging explicitly breaks the $U(4) \times U(4)'$ global symmetry. Under $SU(2)_A \times U(1)_A$, 14 Goldstone bosons — now pseudo-Goldstone bosons (PGBs) — transform as two $\mathbf{2}_{-1/2}$'s and six $\mathbf{1}_0$'s. We identify $SU(2)_A \times U(1)_A$ as $SU(2)_L \times U(1)_Y$ of the standard model. We then find that we can obtain two Higgs doublets as PGBs from this symmetry breaking pattern. The stability of the particular form of the VEVs in eq. (2.2) will be discussed later.

In what sense are the 14 states PGBs? Since the theory is weakly coupled and the gauging of $SU(2)_A \times U(1)_A \times SU(2)_B \times U(1)_B$ explicitly breaks the global $U(4) \times U(4)'$ symmetry by an $O(1)$ amount, the theory does not possess an approximate $U(4) \times U(4)'$. However, as we will see below, radiative corrections from gauge interactions approximately preserve the $U(4) \times U(4)'$ form of the scalar potential, if the discrete Z_2 symmetry interchanging $SU(2)_A$ and $SU(2)_B$, and $U(1)_A$ and $U(1)_B$, is introduced. In this case, $U(4) \times U(4)'$ breaking effects in the scalar potential is of order $1/16\pi^2$, and we can still call the 14 states PGBs.

We now impose the Z_2 symmetry which interchanges the A and B sectors, i.e. H_A and H_B, H_A' and H_B', and the gauge bosons of $SU(2)_A$ and $SU(2)_B$, and $U(1)_A$ and $U(1)_B$. This

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[1]Precisely speaking, the symmetry breaking pattern described by eq. (2.1) is $O(8) \times O(8)' \to O(7) \times O(7)'$, but the existence of these larger symmetries does not affect any of our argument below.

requires the gauge couplings of $SU(2)_A$ and $SU(2)_B$ to be equal, $g_A = g_B = g$, as well as those of $U(1)_A$ and $U(1)_B$, $g'_A = g'_B = g'$. An important consequence of this Z_2 symmetry is that quadratic divergences from gauge loops to the squared-mass parameters for the PGB Higgs bosons are completely eliminated [4]. This is because quadratic divergences appear only in the coefficients of the operators quadratic in fields: $\delta V = \Lambda^2(c_A|H_A|^2 + c_B|H_B|^2 + c'_A|H'_A|^2 + c'_B|H'_B|^2)$, where c_A, c_B, c'_A and c'_B are numbers. (Operators of the form $H_A^\dagger H'_A + \text{h.c.}$ and $H_B^\dagger H'_B + \text{h.c.}$ can be forbidden by imposing a $U(1) \times U(1)'$ global symmetry; see discussion later.) Since the Z_2 symmetry always guarantees that c_A and c_B, and c'_A and c'_B, are equal, quadratically divergent radiative corrections necessarily take the $U(4) \times U(4)'$ invariant form: $\delta V = c_A\Lambda^2(|H_A|^2 + |H_B|^2) + c'_A\Lambda^2(|H'_A|^2 + |H'_B|^2) = c_A\Lambda^2|\Phi|^2 + c'_A\Lambda^2|\Phi'|^2$, which do not give any potential for the PGBs. In fact, one can explicitly check in a nonlinear sigma model that if the gauge couplings of $SU(2)_A$ and $SU(2)_B$, and $U(1)_A$ and $U(1)_B$, are the same as dictated by the Z_2 symmetry, quadratically divergent contributions to the PGB potential are absent. The potential for the PGB Higgs arises from operators of the form

$$\delta V = \xi(|H_A|^4 + |H_B|^4) + \xi'(|H'_A|^4 + |H'_B|^4) + \cdots, \tag{2.3}$$

which are Z_2 invariant but not $U(4) \times U'(4)$ invariant. It is then clear from dimensional analysis that the potential for the PGBs is at most logarithmically divergent.[2] The sizes of the coefficients ξ and ξ' in eq. (2.3) are of order $(g^2/16\pi^2)\ln(\Lambda/f)$, so that the PGB Higgses receive squared masses only of order $(g^2 f^2/16\pi^2)\ln(\Lambda/f)$.

The situation for the Yukawa interactions is similar. If we make the Yukawa couplings Z_2 invariant by introducing mirror quarks, quadratically divergent radiative corrections to the squared masses for the PGB Higgses are eliminated. For example, for the top quarks we introduce mirror quarks $\hat{q}$ and $\hat{\bar{u}}$, which are singlet under $SU(2)_L \times U(1)_Y$ and transform as $\mathbf{2}_{1/6}$ and $\mathbf{1}_{-2/3}$ under $SU(2)_B \times U(1)_B$, in addition to our quarks q and $\bar{u}$, which transform as $\mathbf{2}_{1/6}$ and $\mathbf{1}_{-2/3}$ under $SU(2)_L \times U(1)_Y$ and are singlet under $SU(2)_B \times U(1)_B$. For color interactions, we assume that our quarks and mirror quarks are charged under $SU(3)_A$ and $SU(3)_B$ gauge interactions, respectively, where $SU(3)_A$ is identified as the standard model color group: $SU(3)_A \equiv SU(3)_C$. Writing the Z_2-invariant Yukawa coupling

$$\mathcal{L}_{\text{top}} = y_t(q\bar{u}H_A^\dagger + \hat{q}\hat{\bar{u}}H_B^\dagger), \tag{2.4}$$

the PGBs do not receive any quadratically divergent contributions from this coupling. Here, we couple only $\Phi = (H_A|H_B)$ to the top quarks, and not $\Phi' = (H'_A|H'_B)$. Such a situation can be easily arranged, for example, by considering that the $U(1) \times U(1)'$ subgroup of the $U(4) \times U(4)'$ global symmetry is an exact (anomalous) global symmetry and assigning appropriate charges to the quark fields. The symmetry $U(1) \times U(1)'$ will also be discussed later when we introduce an explicit $U(4) \times U(4)'$ breaking operator in the scalar potential.

There are two ways to introduce the bottom Yukawa coupling into the theory, without introducing dangerous flavor changing neutral currents. One way is to couple only $\Phi =$

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[2]Radiative corrections involving higher dimension operators can, of course, generate power divergent corrections to the PGB potential, but they are sufficiently small if the theory is weakly coupled at Λ.

$(H_A|H_B)$ to the bottom quarks:

$$\mathcal{L}_{\text{bottom}} = y_b(q\bar{d}H_A + \hat{q}\hat{\bar{d}}H_B), \tag{2.5}$$

where $\bar{d}$ is the right-handed bottom quark transforming as $\mathbf{1}_{1/3}$ under $\text{SU}(2)_L \times \text{U}(1)_Y$ and singlet under $\text{SU}(2)_B \times \text{U}(1)_B$, while $\hat{\bar{d}}$ is its mirror partner transforming as $\mathbf{1}_{1/3}$ under $\text{SU}(2)_B \times \text{U}(1)_B$ and singlet under $\text{SU}(2)_L \times \text{U}(1)_L$. The other way is to couple only $\Phi' = (H'_A|H'_B)$ to the bottom quarks:

$$\mathcal{L}_{\text{bottom}} = y_b(q\bar{d}H'_A + \hat{q}\hat{\bar{d}}H'_B). \tag{2.6}$$

Since our two PGB-Higgs doublets essentially come from H_A and H'_A, the two cases of eqs. (2.5) and (2.6) lead, respectively, to Type-I and Type-II Higgs doublet theories. The Yukawa couplings for lighter quarks can be obtained by making y_t and y_b to 3×3 matrices. The Yukawa couplings for leptons are introduced analogously to the down-type quarks, but the choice between eqs. (2.5) and (2.6) can be made independently from that for the down-type quarks. The particular form of the couplings in eq. (2.5) or (2.6) can, again, be ensured by considering that the $\text{U}(1) \times \text{U}(1)'$ subgroup of $\text{U}(4) \times \text{U}(4)'$ is exact and by assigning appropriate $\text{U}(1) \times \text{U}(1)'$ charges to q, $\bar{d}$, $\hat{q}$ and $\hat{\bar{d}}$ (and to the corresponding lepton fields).

With these structures for gauge and Yukawa interactions, radiative corrections to the squared masses for the PGB Higgses can be made small to the level of $O((f^2/16\pi^2)\ln(\Lambda/f))$. This itself, however, does not achieve our goal of naturally raising the cutoff Λ to the multi-TeV region. Since our Higgs fields, h, are PGBs, their potential generated by gauge and Yukawa interactions is a function of $\cos(h/f)$ and $\sin(h/f)$, giving $\langle h \rangle \approx f \approx 200$ GeV. This in turn implies $\Lambda \lesssim 4\pi f \approx 2$ TeV. The source of the problem is that while the Higgs mass-squared parameters are suppressed to the level of $O((f^2/16\pi^2)\ln(\Lambda/f))$, the quartic couplings are also suppressed and of order $O((1/16\pi^2)\ln(\Lambda/f))$. Moreover, the stability of the particular form of the VEVs in eq. (2.2) is not obvious at this stage, without a detailed study of the PGB potential generated at loop level.

We now present a mechanism addressing these issues and present a realistic theory in which Λ can be raised to the multi-TeV region without a significant fine-tuning. Suppose we introduce a tree-level operator

$$\mathcal{L}_H = -\lambda(|H_A^\dagger H'_A|^2 + |H_B^\dagger H'_B|^2), \tag{2.7}$$

which explicitly violates the global $\text{U}(4) \times \text{U}(4)'$ symmetry but preserves the Z_2 symmetry. We take the coupling λ to be of $O(1)$. We then find that the operator of eq. (2.7) gives an order-one quartic coupling for the two PGB-Higgs doublets without giving large squared masses, and at the same time stabilizes the desired vacuum of eq. (2.2). To see this explicitly, we expand the Φ and Φ' fields as

$$\Phi = \begin{pmatrix} H_A \\ \hline H_B \end{pmatrix} = \exp\left[\frac{i}{f}\begin{pmatrix} 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & h_2 \\ 0 & 0 & 0 & \frac{a+ib}{\sqrt{2}} \\ h_1^\dagger & h_2^\dagger & \frac{a-ib}{\sqrt{2}} & \frac{c}{\sqrt{2}} \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix}, \tag{2.8}$$

and

$$\Phi' = \begin{pmatrix} H'_A \\ H'_B \end{pmatrix} = \exp\left[\frac{i}{f'}\begin{pmatrix} 0 & 0 & h'_1 & 0 \\ 0 & 0 & h'_2 & 0 \\ h'^\dagger_1 & h'^\dagger_2 & \frac{c'}{\sqrt{2}} & \frac{a'-ib'}{\sqrt{2}} \\ 0 & 0 & \frac{a'+ib'}{\sqrt{2}} & 0 \end{pmatrix}\right]\begin{pmatrix} 0 \\ 0 \\ f' \\ 0 \end{pmatrix}, \tag{2.9}$$

respectively, where $h = (h_1, h_2)$ and $h' = (h'_1, h'_2)$ are the two PGB Higgs doublets, and a, b, c, a', b' and c' are the six singlet PGBs, of the spontaneous $U(4) \times U(4)' \to U(3) \times U(3)'$ breaking. Here the PGB fields are canonically normalized, and we have neglected the radial excitation modes of Φ and Φ'. Substituting eqs. (2.8), (2.9) into eq. (2.7), we obtain

$$V = -\mathcal{L}_H = \lambda|h^\dagger h'|^2 + \frac{1}{2}\lambda(f^2 + f'^2)(\tilde{a}^2 + \tilde{b}^2) + \cdots, \tag{2.10}$$

where $\tilde{a} \equiv (f'a - fa')/\sqrt{f^2 + f'^2}$ and $\tilde{b} \equiv (f'b + fb')/\sqrt{f^2 + f'^2}$ are canonically normalized singlet PGBs that are not eaten by the massive $SU(2)_B \times U(1)_B$ gauge bosons. (The eaten modes are $(fa + f'a')/\sqrt{f^2 + f'^2}$, $(fb - f'b')/\sqrt{f^2 + f'^2}$, c and c'.) This explicitly shows that for $\lambda > 0$ the operator of eq. (2.7) gives a quartic coupling λ to the two PGB-Higgs doublets, h and h', and the vacuum of eq. (2.2) is stabilized by the masses of $\tilde{a}$ and $\tilde{b}$, $m_{\tilde{a}}^2 = m_{\tilde{b}}^2 = \lambda(f^2 + f'^2) > 0$. A similar operator has also been used in little Higgs theories to obtain a tree-level quartic coupling [9].

An interesting point here is that while the operator of eq. (2.7) introduces an order-one explicit breaking of the global $U(4) \times U(4)'$ symmetry to the scalar potential (and hence the $O(f)$ masses for $\tilde{a}$ and $\tilde{b}$), the Higgs doublets do not obtain masses of order f. The masses are generated at loop level, but because of Z_2 invariance they are generated only through *quartic* couplings between H's, such as the ones in eq. (2.3). The coefficients of these operators, e.g. ξ and ξ' in eq. (2.3), are at most of order $(1/16\pi^2)\ln(\Lambda/f)$, since they are generated at loop level and the theory is weakly coupled.[3] This guarantees that radiatively generated Higgs squared masses cannot be larger than of order $(f^2/16\pi^2)\ln(\Lambda/f)$. We note here that our 14 states are no longer "PGBs" in the usual sense, since the potential as a whole does not possess an approximate $U(4) \times U(4)'$ symmetry: it is broken by an $O(1)$ amount by λ. What ensures the stability of the potential here under radiative corrections from explicit symmetry breaking interactions is the "partial $U(4) \times U(4)'$ symmetry" — $U(4) \times U(4)'$ possessed only by the quadratic terms of the scalar potential, which arises as a consequence of the discrete Z_2 symmetry of the theory.

It is technically natural to introduce only the operator of eq. (2.7) as an $O(1)$ $U(4) \times U(4)'$-violating effect in the scalar potential. Other $U(4) \times U(4)'$-violating terms are generated at loop level, but they are at most of order $1/16\pi^2$. In fact, this particular explicit symmetry breaking pattern may be justified by assuming certain structure for the ultraviolet theory above Λ. Imagine, for example, that the operator of eq. (2.7) is generated by tree-level exchanges of auxiliary scalar fields that have $Z_2 \times U(1) \times U(1)'$ invariant

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[3]The argument here shows that the theory could potentially have a problem if it is strongly coupled e.g. $\eta \sim \eta' \sim 4\pi$, because then the coefficients ξ and ξ' may receive corrections of order e.g. $\eta\lambda/16\pi^2$, $\eta g^2/16\pi^2 \sim 1$, giving the Higgs squared masses of order f^2, which would be too large for our purpose [10]. We will discuss this issue in section 5.

trilinear couplings between primed, non-primed and the auxiliary fields. Then, the only $U(4) \times U(4)'$-violating operators generated at tree level are the one in eq. (2.7) and an operator $H_A^\dagger H_A' H_B H_B'^\dagger + \text{h.c.}$ We find that the existence of the latter operator with an $O(1)$ coefficient does not change any of the basic aspects of the model. This operator, however, can also be forbidden if we impose a discrete Z_2 "chiral" symmetry: $(H_A|H_B) \leftrightarrow (H_A|H_B)$ and $(H_A'|H_B') \leftrightarrow (H_A'|-H_B')$. Below we impose this Z_2 symmetry and set the coefficient of the above operator to be zero for simplicity. We also impose the $U(1) \times U(1)' \subset U(4) \times U(4)'$ symmetry as an exact (anomalous) global symmetry of the model. This suppresses the operator $\Phi^\dagger \Phi' + \text{h.c.}$, whose coefficient must be of order f^2 or smaller since otherwise some of the modes needed to cancel quadratic divergences to the Higgs squared masses become too heavy.

Summarizing so far, the lagrangian of our theory is given by the scalar potential of eqs. (2.1), (2.7) and the Yukawa couplings of either eqs. (2.4), (2.5) or eqs. (2.4), (2.6). At scales below f, the theory contains the standard model quarks and leptons as well as the two Higgs doublets, h and h', which have the following dimensionless couplings:

$$\mathcal{L} = y_t q \bar{u} h^\dagger + y_b q \bar{d} h^{(\prime)} + y_\tau l \bar{e} h^{(\prime)} - V(h, h'), \tag{2.11}$$

where l and $\bar{e}$ are the doublet and singlet lepton fields, respectively, and the Higgs potential $V(h, h')$ contains the tree-level quartic coupling $\lambda |h^\dagger h'|^2$ as well as radiatively generated Higgs mass-squared parameters of order $(f^2/16\pi^2) \ln(\Lambda/f)$. The Higgs field to which the down-type quarks and charged leptons couple can be either h or h', depending on their $U(1) \times U(1)'$ charges, and the choice can be made independently for the down-type quarks and charged leptons (if there is no quark-lepton unification in the fundamental theory).

Because of the particular form of the Higgs quartic coupling arising from the operator of eq. (2.7), $\lambda |h^\dagger h'|^2$, the squared mass parameters for h and h' must both be positive to ensure the absence of a runaway direction in the potential. Electroweak symmetry breaking then must be caused by the term $h^\dagger h' + \text{h.c.}$, by making one of the eigenvalues in the Higgs mass-squared matrix negative. We assume that these mass terms are generated by soft Z_2-symmetry breaking operators

$$\mathcal{L}_{\text{soft}} = -\mu^2 |H_A|^2 - \mu'^2 |H_A'|^2 + (b H_A^\dagger H_A' + \text{h.c.}), \tag{2.12}$$

where we take parameters μ^2, μ'^2 and b to be of order $(f^2/16\pi^2) \ln(\Lambda/f)$, which is technically natural. The Higgs potential $V(h, h')$ is then given by

$$V(h, h') = m^2 |h|^2 + m'^2 |h'|^2 - (b h^\dagger h' + \text{h.c.}) + \lambda |h^\dagger h'|^2, \tag{2.13}$$

where m^2 and m'^2 are given at tree level by μ^2 and μ'^2, respectively, but they also receive radiative corrections of order $(f^2/16\pi^2) \ln(\Lambda/f)$. Here, we have suppressed radiatively generated quartic terms as well as higher order terms. The conditions for having the stable minimum breaking the electroweak symmetry are

$$m^2 > 0, \qquad m'^2 > 0, \qquad |b|^2 > m^2 m'^2. \tag{2.14}$$

With these conditions satisfied, we expect to obtain the desired hierarchy

$$\Lambda \approx 4\pi f \approx (4\pi)^2 v, \tag{2.15}$$

without a significant fine-tuning, where $v \equiv (\langle h \rangle^2 + \langle h' \rangle^2)^{1/2} \simeq 174$ GeV. To reliably estimate how large we can make Λ without fine-tuning, however, we need to calculate radiative correction to m^2 and m'^2 from top-Yukawa, gauge and Higgs-quartic interactions, and carefully study fine-tuning required to obtain successful electroweak symmetry breaking. This will be performed in the next section, where we find that the estimate of eq. (2.15) is somewhat too optimistic.

3. Analysis of fine-tuning

Since radiative corrections to the Higgs mass squared parameters in our theory come only from the *quartic* terms in the scalar potential, we can reliably estimate their sizes at the leading-log level. Specifically, given the lagrangian of eqs. (2.1), (2.4), (2.7), we can evaluate the coefficients of U(4) × U(4)′-violating operators

$$\mathcal{L} = -\xi(|H_A|^4 + |H_B|^4) - \xi'(|H'_A|^4 + |H'_B|^4) - \kappa(|H_A|^2|H'_A|^2 + |H_B|^2|H'_B|^2), \tag{3.1}$$

that give masses for the Higgs doublets, where we have kept only operators that preserve U(1) × U(1)′ and the "chiral" Z_2 symmetry. We can then obtain expressions for radiative corrections to the Higgs mass-squared parameters in terms of the renormalized f and f' parameters. This determines how large we can make f and f' without severe fine-tuning, which in turn determines how large the cutoff scale Λ can be. In our analysis we assume either that down-type quarks and leptons couple to h or that the ratio $\langle h \rangle / \langle h' \rangle$ is not very large, so that only the relevant Yukawa coupling is the top Yukawa coupling. An extension to include the bottom and tau Yukawa couplings, however, is straightforward.

At the one-loop leading-log level, the coefficients ξ, ξ' and κ in eq. (3.1) receive the following radiative corrections:

$$\delta\xi = \frac{1}{16\pi^2}\left(6y_t^4 - \frac{9}{8}g^4 - \frac{3}{4}g^2g'^2 - \frac{3}{8}g'^4 - \lambda^2\right)\ln\frac{\Lambda}{f}, \tag{3.2}$$

$$\delta\xi' = \frac{1}{16\pi^2}\left(-\frac{9}{8}g^4 - \frac{3}{4}g^2g'^2 - \frac{3}{8}g'^4 - \lambda^2\right)\ln\frac{\Lambda}{f}, \tag{3.3}$$

$$\delta\kappa = \frac{1}{16\pi^2}\left(-\frac{9}{4}g^4 + \frac{3}{2}g^2g'^2 - \frac{3}{4}g'^4 - 2\lambda^2\right)\ln\frac{\Lambda}{f}, \tag{3.4}$$

where y_t is the top Yukawa coupling in eq. (2.4), η, η' and λ are couplings in eqs. (2.1), (2.7), g is the Z_2 invariant gauge coupling of SU(2)$_A \equiv$ SU(2)$_L$ and SU(2)$_B$, and g' that of U(1)$_A \equiv$ U(1)$_Y$ and U(1)$_B$. The finite pieces depend on the unknown ultraviolet theory and do not have a real physical meaning in the effective theory. From these equations, we obtain the expressions for the corrections to the Higgs mass-squared parameters m^2, m'^2 and b in eq. (2.13):

$$\delta m^2 = -2f^2\,\delta\xi - f'^2\,\delta\kappa, \tag{3.5}$$

$$\delta m'^2 = -2f'^2\,\delta\xi' - f^2\,\delta\kappa, \tag{3.6}$$

$$\delta b = 0, \tag{3.7}$$

which are of order $(f^2/16\pi^2)\ln(\Lambda/f)$. Contributions arising from renormalizations of the μ^2 and μ'^2 parameters in eq. (2.12) are of order $(f^2/(16\pi^2)^2)\ln(\Lambda/f)$ and thus negligible.

What is the amount of fine-tuning for this potential? Let us first see that the fine-tuning parameter Δ^{-1} can be represented in terms of the lagrangian parameters and/or physical Higgs boson masses in the following way [11]. The equations determining the minimum of the potential, eq. (2.13), can be written as

$$\tan^2\beta = \frac{m'^2}{m^2}, \tag{3.8}$$

$$\lambda v^2 = \frac{2b}{\sin 2\beta} - (m^2 + m'^2), \tag{3.9}$$

where $\tan\beta \equiv \langle h \rangle / \langle h' \rangle$ and $v \equiv (\langle h \rangle^2 + \langle h' \rangle^2)^{1/2} \simeq 174$ GeV is the electroweak scale. We then find that the only source of a potential unnatural cancellation is in the right-hand-side of eq. (3.9), and that the fine-tuning parameter Δ^{-1} is approximately given by the ratio of λv^2 and $m^2 + m'^2$ (or $2b/\sin 2\beta$): $\Delta^{-1} \approx \lambda v^2/(m^2 + m'^2)$. (Note that m^2 and m'^2 are both positive, so that they cannot be canceled with each other.) On the other hand, the masses of the physical Higgs bosons are given by

$$m_{A^0}^2 = m^2 + m'^2 + \lambda v^2, \tag{3.10}$$

$$m_{H^\pm}^2 = m^2 + m'^2, \tag{3.11}$$

$$m_{H^0,h^0}^2 = \frac{1}{2}\left\{ m_{A^0}^2 \pm \sqrt{m_{A^0}^4 \cos^2 2\beta + (m_{A^0}^2 - 2\lambda v^2)^2 \sin^2 2\beta} \right\}, \tag{3.12}$$

where A^0, $H^\pm$, H^0, and h^0 represent the pseudoscalar, charged, heavier neutral, and lighter neutral Higgs bosons, respectively. Assuming that the lighter neutral Higgs boson h^0 is somewhat lighter than the other Higgs bosons, we obtain

$$m_{H^0}^2 \simeq m_{A^0}^2 = m^2 + m'^2 + \lambda v^2, \tag{3.13}$$

$$m_{H^\pm}^2 = m^2 + m'^2, \tag{3.14}$$

$$m_{h^0}^2 \simeq \lambda v^2 \sin^2 2\beta. \tag{3.15}$$

The fine-tuning parameter can then be written as

$$\Delta^{-1} \approx \frac{\lambda v^2}{m^2 + m'^2} \simeq \frac{m_{h^0}^2}{m_{H^\pm}^2 \sin^2 2\beta}. \tag{3.16}$$

For $\tan\beta$ not much larger than 1, e.g. $\tan\beta \lesssim 2$, this simplifies further to $\Delta^{-1} \sim m_{h^0}^2/m_{H^\pm}^2$.

We now estimate how high we can push up the cutoff scale Λ. Here we assume $f \simeq f'$ for simplicity. First, we rewrite eq. (3.16) using eqs. (3.8), (3.15) as $\Delta^{-1} \simeq \lambda v^2/((1 + \tan^2\beta)m^2) \simeq m_{h^0}^2/(4m^2 \sin^2\beta)$. The parameter m^2 receives contributions both at tree level, $m^2|_{\text{tree}} = \mu^2$, and at radiative level, δm^2 in eq. (3.5). In order to avoid unnatural cancellations among these contributions, m^2 itself must be at least of the same size as the largest radiative contribution. For $f \simeq f'$, the largest one comes either from the top loop contribution:

$$\delta m^2|_{\text{top}} = -\frac{3y_t^4}{4\pi^2}f^2 \ln\frac{\Lambda}{f}, \tag{3.17}$$

where we have used eqs. (3.2), (3.5), or from the Higgs quartic contribution:

$$\delta m^2|_{H^4} = \frac{\lambda^2}{8\pi^2}(f^2 + f'^2)\ln\frac{\Lambda}{f}, \tag{3.18}$$

where we have used eqs. (3.2), (3.4), (3.5). Now, setting $m^2 \approx |\delta m^2|_{\text{top}}|$ and using $m_t = y_t v \sin\beta$, the contribution to the fine-tuning parameter from top loop can be written as:

$$\Delta^{-1}|_{\text{top}} \equiv \frac{m_{h^0}^2}{4|\delta m^2|_{\text{top}}|\sin^2\beta} \approx \frac{\pi^2 v^4 m_{h^0}^2 \sin^2\beta}{3m_t^4 f^2 \ln(\Lambda/f)} \simeq 2\frac{m_{h^0}^2 \sin^2\beta}{f^2}, \tag{3.19}$$

where we have used $m_t = m_t|_{\text{pole}}(1 + g_3^2/3\pi^2)^{-1} \simeq 166$ GeV and $\ln(\Lambda/f) \simeq \ln(2\pi)$ in the last equation (see below). The contribution from quartic loop, on the other hand, can be written using eq. (3.15) as

$$\Delta^{-1}|_{H^4} \equiv \frac{m_{h^0}^2}{4\delta m^2|_{H^4}\sin^2\beta} \approx \frac{32\pi^2 v^4 \sin^2\beta \cos^4\beta}{m_{h^0}^2(f^2 + f'^2)\ln(\Lambda/f)} \simeq \frac{\sin^2\beta \cos^4\beta}{m_{h^0}^2 f^2}(530 \text{ GeV})^4, \tag{3.20}$$

where we have set $f = f'$ in the last equation. The fine-tuning parameter Δ^{-1} is then given by

$$\Delta^{-1} = \min\left\{\Delta^{-1}|_{\text{top}}, \ \Delta^{-1}|_{H^4}\right\}. \tag{3.21}$$

From eqs. (3.19), (3.20), (3.21), we find that a maximum value for Δ^{-1} is obtained for $m_{h^0}^2 \simeq (530 \text{ GeV})^2 \cos^2\beta/\sqrt{2}$, with the value $\Delta^{-1} \simeq (320 \text{ GeV}/f)^2 \sin^2 2\beta$. Under the constraint from precision electroweak measurements, $m_{h^0} \lesssim 250$ GeV [12], this occurs when $m_{h^0} \simeq 250$ GeV and $\tan\beta \simeq 1.5$, and the largest value of f for a fixed Δ^{-1} is given by

$$f_{\max} \approx 650 \text{ GeV}\left(\frac{20\%}{\Delta^{-1}}\right)^{1/2}. \tag{3.22}$$

(This value can also be reproduced by taking a complete one-loop effective potential into account and seeing the sensitivity of v^2 with respect to the parameter b.) The relation between Λ and f is not calculable because f^2 receives quadratically divergent radiative corrections proportional to Λ^2. The relation, however, can be estimated using a naive scaling argument:

$$f^2 \approx \frac{N_f}{16\pi^2}\Lambda^2, \tag{3.23}$$

where N_f is the number of "flavors", which is 4 in our case. For $\eta \gtrsim 1$, this relation roughly agrees with the result obtained naively by calculating the coefficient of the quadratic divergence of f^2 in the effective theory. For smaller η, the hierarchy between Λ and f may be smaller because of the top Yukawa contribution to the Φ mass term. From eqs. (3.22), (3.23) we obtain the maximum value of the cutoff:

$$\Lambda_{\max} \approx 4 \text{ TeV}\left(\frac{20\%}{\Delta^{-1}}\right)^{1/2}. \tag{3.24}$$

To evade the experimental constraints from higher dimension operators we need to have $\Lambda \gtrsim 5$ TeV.[4] Our theory requires (only) a mild fine-tuning of about

$$\Delta^{-1} \approx 14\% \left(\frac{5 \text{ TeV}}{\Lambda}\right)^2, \tag{3.25}$$

to achieve this. If we restrict ourselves to $m_{h^0} \lesssim 200$ GeV, this number becomes $\approx 10\%$. We note here that the precise number in eq. (3.25) is subject to uncertainties of order $(20\sim30)\%$, for example, due to finite corrections at Λ to δm^2, $\delta m'^2$ and f^2.

We find that our theory does not really give the naive hierarchies of eq. (2.15). This is because for large $\tan\beta$, we need to have m'^2 much larger than m^2 (see eq. (3.8)), so that we need to cancel this large m'^2 with the b term in the minimization equation of eq. (3.9). The Higgs quartic coupling λ also becomes large in this region, and fine-tuning from this parameter, $\Delta^{-1}|_{H^4}$, also becomes severe. For smaller $\tan\beta$, on the other hand, the top Yukawa coupling becomes larger, making fine-tuning from top loop, $\Delta^{-1}|_{\text{top}}$, worse. This is especially the case because the top radiative correction to m^2 is proportional to y_t^4 (see eq. (3.17)). Here the extra y_t^2 in addition to the naive y_t^2 comes from the fact that the particle cutting off the top divergence in the standard model is the mirror top quark, whose mass is proportional to y_t: $m_{\hat{q}} = y_t f$. In the next section we present a theory in which the contribution from the standard model top quark is canceled by the U(4) partner of the top quark, in which case the logarithmic sensitivity of the top contribution to Λ is eliminated and we can achieve further reduction of fine-tuning (or push up Λ further for a given Δ^{-1}).

To assess the degree of success here, let us compare our theory with the standard model (with the tree-level Higgs mass-squared parameter set to zero by hand). In the standard model, the Higgs mass-squared parameter receives quadratically divergent contribution, whose cutoff will in general be different from that of f^2 in eq. (3.23). It is, therefore, not possible to make a real comparison between the two theories. Nevertheless, if we naively take the quadratic divergent part from the top loop, $\delta m_h^2 = -(3y_t^2/8\pi^2)\Lambda^2$, and simply define the fine-tuning parameter for the standard model by $\Delta_{\text{SM}}^{-1} = \lambda v^2/|\delta m_h^2|$, the standard model gives

$$\Delta_{\text{SM}}^{-1} \approx 3.5\% \left(\frac{5 \text{ TeV}}{\Lambda}\right)^2, \tag{3.26}$$

under the same constraint of $m_{h^0} \lesssim 250$ GeV (our definitions for m_h^2 and λ are $V(h) = m_h^2|h|^2 + (\lambda/2)|h|^4$). For $m_{h^0} \lesssim 200$ GeV, this number becomes $\approx 2.3\%$.

Equations (3.25), (3.26) imply that our theory achieves about a factor 4 reduction in fine-tuning. For $\Lambda \approx 5$ TeV, the scale relevant for electroweak precision constraints, the fine-tuning goes from "a few percent" to "better than 10%" for $m_{h^0} \gtrsim 200$ GeV (about 1 in 7 for $m_{h^0} \simeq 250$ GeV). We also note that some of the factors included in the analysis here, for example N_f in eq. (3.23), are often not included in literature. To compare the result of our model with those of other models, we must take all these factors into account appropriately.

---

[4]Some higher dimension operators, e.g. $h^\dagger \sigma^a h W_{\mu\nu}^a B^{\mu\nu}$, require $\Lambda \simeq 10$ TeV if the coefficients are really 1. They are, however, expected to carry factors of, e.g., gg' in front, in which case the bound on Λ is somewhat weaker and of order several TeV.

In the next section, we extend the minimal theory presented here to include the U(4) partners of the top quark. This allows a further reduction of fine-tuning and/or a larger cutoff scale, since the top contribution to the divergence of the Higgs mass-squared parameter is then canceled by these partners.

4. U(4)-invariant top sector

In this section we extend the top quark sector of the previous model to include the U(4) partners of the top quark. Following ref. [4], we promote the left-handed top quark, q, and its mirror partner, $\hat{q}$, into the U(4)-invariant field:

$$
\begin{aligned}
Q = {} & q(\mathbf{3}, \mathbf{2}, 1/6; \mathbf{1}, \mathbf{1}, 0) + \hat{q}(\mathbf{1}, \mathbf{1}, 0; \mathbf{3}, \mathbf{2}, 1/6) \\
& + q'(\mathbf{3}, \mathbf{1}, 2/3; \mathbf{1}, \mathbf{2}, -1/2) + \hat{q}'(\mathbf{1}, \mathbf{2}, -1/2; \mathbf{3}, \mathbf{1}, 2/3),
\end{aligned}
\tag{4.1}
$$

where the numbers in parentheses represent gauge quantum numbers under $(\mathrm{SU}(3)_A \times \mathrm{SU}(2)_A \times \mathrm{U}(1)_A) \times (\mathrm{SU}(3)_B \times \mathrm{SU}(2)_B \times \mathrm{U}(1)_B)$. The q' and $\hat{q}'$ are new fields introduced in this procedure. Defining the field representing the right-handed top quark, $\bar{u}$, and its mirror partner, $\hat{\bar{u}}$, as

$$
\bar{U} = \bar{u}(\mathbf{3^*}, \mathbf{1}, -2/3; \mathbf{1}, \mathbf{1}, 0) + \hat{\bar{u}}(\mathbf{1}, \mathbf{1}, 0; \mathbf{3^*}, \mathbf{1}, -2/3),
\tag{4.2}
$$

we can write the following U(4) ($\times$ U(4)$'$) invariant top Yukawa coupling:

$$
\mathcal{L}_{\mathrm{top}} = y_t Q \bar{U} \Phi^\dagger,
\tag{4.3}
$$

which contains the Yukawa couplings of eq. (2.4) when expanded in the "component" fields of eqs. (4.1), (4.2). The new fields q' and $\hat{q}'$ in eq. (4.1) are made heavy by introducing the conjugate fields $q'^c(\mathbf{3^*}, \mathbf{1}, -2/3; \mathbf{1}, \mathbf{2}, 1/2)$ and $\hat{q}'^c(\mathbf{1}, \mathbf{2}, 1/2; \mathbf{3^*}, \mathbf{1}, -2/3)$ with the Z_2-invariant mass term

$$
\mathcal{L} = M(q' q'^c + \hat{q}' \hat{q}'^c).
\tag{4.4}
$$

With the new top Yukawa coupling of eq. (4.3), the only U(4) $\times$ U(4)$'$ violating effect in the top sector is the mass M of eq. (4.4). The contribution from the top quark to the Higgs mass-squared parameter is thus cut off at the scale M, which we take $\approx y_t f$.

The calculation of radiative corrections from the Q and $\bar{U}$ fields to the Higgs mass-squared parameter has been performed in [4]. In the present context, this translates into

$$
\delta m^2|_{\mathrm{top}} = -\frac{3}{8\pi^2} \frac{y_t^2 M^2}{y_t^2 f^2 - M^2} \left(M^2 \ln \frac{y_t^2 f^2 + M^2}{M^2} - y_t^2 f^2 \ln \frac{y_t^2 f^2 + M^2}{y_t^2 f^2} \right),
\tag{4.5}
$$

and the top quark mass is given by

$$
m_t \simeq \frac{y_t M}{\sqrt{y_t^2 f^2 + M^2}} \, v \sin\beta.
\tag{4.6}
$$

The top contribution of eq. (4.5) can be written in the form

$$
\delta m^2|_{\mathrm{top}} = -\frac{3}{8\pi^2} y_t^2 M^2 \, \mathcal{F}\!\left(\frac{y_t^2 f^2}{M^2} \right),
\tag{4.7}
$$

where $\mathcal{F}(x) \equiv \{\ln(1 + x) - x\ln(1 + 1/x)\}/(x - 1)$ is a function which has the property $\mathcal{F}(x) = \mathcal{F}(1/x)$. For $0.5 \lesssim x \lesssim 2$, this function takes values $\mathcal{F}(x) \simeq 0.3$. We then find that, in the parameter region $0.5 \lesssim y_t^2 f^2/M^2 \lesssim 2$, the top contribution in the present model, eq. (4.5), is a factor of $(2 \sim 3)$ smaller than that in the previous model, eq. (3.17), for the same value of f. The contribution to the fine-tuning parameter from top loop, which is given by $\Delta^{-1}|_{\text{top}} \approx m_{h^0}^2/(4|\delta m^2|_{\text{top}}|\sin^2\beta)$, is thus a factor of $(2 \sim 3)$ smaller than before. The contribution from quartic loop, $\Delta^{-1}|_{H^4}$, is the same and is given by eq. (3.20). The fine-tuning parameter Δ^{-1} is given by the smaller of $\Delta^{-1}|_{\text{top}}$ and $\Delta^{-1}|_{H^4}$, as in eq. (3.21).

We can now repeat the same analysis as in the previous model with the new $\Delta^{-1}|_{\text{top}}$. We find that under the constraint $m_{h^0} \lesssim 250$ GeV, the largest value of f for a fixed Δ^{-1} is given by

$$f_{\max} \approx 930 \text{ GeV} \left(\frac{20\%}{\Delta^{-1}}\right)^{1/2}, \tag{4.8}$$

which occurs when $m_{h^0} \simeq 250$ GeV, $y_t \sim \lambda \sim 2$, $\tan\beta \sim 1$ and $M \simeq y_t f$. (For $M \neq y_t f$ with the other parameters fixed, $\Delta^{-1} \propto x/((x + 1)^2 \mathcal{F}(x))$ where $x \equiv y_t^2 f^2/M^2$, so that Δ^{-1} changes only $\lesssim 20\%$ for $0.5 \lesssim y_t f/M \lesssim 2$.) For $m_{h^0} \lesssim 200$ GeV, this number becomes ≈ 890 GeV, occurring at $m_{h^0} \simeq 200$ GeV, $y_t \sim \lambda \sim 1.5$, $\tan\beta \simeq 1.4$ and $M \simeq y_t f$. Since $\Lambda \approx 2\pi f$, we obtain the maximum value of the cutoff:

$$\Lambda_{\max} \approx 6 \text{ TeV} \left(\frac{20\%}{\Delta^{-1}}\right)^{1/2}. \tag{4.9}$$

For $\Delta^{-1} \simeq 10\%$, this reaches as high as $\Lambda_{\max} \approx 8$ TeV. In terms of the fine-tuning parameter, we find

$$\Delta^{-1} \approx 28\% \left(\frac{5 \text{ TeV}}{\Lambda}\right)^2. \tag{4.10}$$

Compared with the standard model case, eq. (3.26), this is an improvement of a factor ≈ 8. This is achieved because the top contribution to the Higgs mass-squared parameter is cut off at the scale $M \approx y_t f$, without a logarithmic sensitivity to Λ.

5. Possibility of strong coupling at Λ

In previous sections we have assumed that the theory is weakly coupled at Λ. This has ensured that radiative corrections to the $U(4) \times U(4)'$-violating quartic terms from the gauge, Yukawa and λ couplings are of order $(1/16\pi^2)\ln(\Lambda/f)$. If the theory is strongly coupled at Λ, i.e. $\eta \sim \eta' \sim 16\pi^2$, this property is not automatically guaranteed, since the couplings may, in general, receive corrections of order e.g. $\eta g^2/16\pi^2, \eta\lambda/16\pi^2 \sim 1$ [10]. In this section we discuss the possibility of making the theory strongly coupled at Λ.

Let us first consider the corrections from the coupling λ in eq. (2.7). We find that we can rewrite the operator in eq. (2.7) as

$$\mathcal{L}_H = -\lambda_1 |H_A^\dagger H_A' + H_B^\dagger H_B'|^2 - \lambda_2 |H_A^\dagger H_A' - H_B^\dagger H_B'|^2, \tag{5.1}$$

where $\lambda_1 = \lambda_2 = \lambda/2$. We then find that the first term preserves a $U(4)$ global symme-

try under which $(H_A|H_B)$ and $(H'_A|H'_B)$ transform as a fundamental representation, while the second term preserves another U(4) symmetry under which $(H_A|H_B)$ and $(H'_A|-H'_B)$ transform as a fundamental representation. Each of these U(4)'s is sufficient to protect the mass of the Higgs fields h and h', implying that the dangerous operators in eq. (3.1) are generated only at order $\lambda_1\lambda_2 \sim \lambda^2$. This guarantees that these operators receive radiative corrections only of order $16\pi^2(\lambda/16\pi^2)^2\ln(\Lambda/f) \sim (1/16\pi^2)\ln(\Lambda/f)$ even at strong coupling (with the explicit breaking parameter λ kept to be $O(1)$, of course). We find that the collective symmetry breaking mechanism [2] is automatically incorporated in the single operator of eq. (2.7) in our theory.[5]

We next consider the Yukawa couplings. The Yukawa couplings, collectively denoted as y here, connect two fermions to a scalar field Φ or Φ'. At order y^2, only the quadratic terms in the scalar potential receive corrections. These terms, however, are necessarily U(4) × U(4)$'$ invariant due to the Z_2 symmetry and the global U(1) × U(1)$'$ symmetry. The corrections to the Higgs mass-squared parameters, which come from the quartic terms in the scalar potential, thus arise only at order y^4. This ensures that the dangerous operators receive corrections only of order $16\pi^2(y^2/16\pi^2)^2\ln(\Lambda/f) \sim (1/16\pi^2)\ln(\Lambda/f)$ even at strong coupling.

How about gauge interactions? At the renormalizable level, we can show that the dangerous corrections do not arise, analogously to the case of the λ coupling. First, interactions of the form $\Phi^\dagger\Phi A^\mu A_\mu$ generate only the quadratic terms in the scalar potential, but they are always U(4) × U(4)$'$ invariant. The interactions linear in A_μ can then be decomposed into two parts, as in eq. (5.1), each of which preserves a global U(4) symmetry that is sufficient to protect the masses of h and h'. The dangerous operators thus receive corrections only of order $16\pi^2(g^2/16\pi^2)^2\ln(\Lambda/f) \sim (1/16\pi^2)\ln(\Lambda/f)$. This argument, however, may not apply for higher dimension operators suppressed by Λ, which we expect to be there. Showing that the theory can really be made strongly coupled at Λ, therefore, requires a careful analysis of all these corrections. Here we do not pursue this issue further, leaving it for future work [13].

The possibility of strong coupling at Λ is particularly interesting because the relation $f \approx f' \approx \Lambda/2\pi$ would then be naturally understood in terms of naive dimensional analysis [14]. It also leads to more possibilities for an ultraviolet theory above the scale Λ. A closer study of this issue is warranted.

6. Phenomenology

In this section we discuss the phenomenological implications of our models. We mainly focus on collider signals, since the cosmological aspects of the models are similar to what were discussed in literature (see e.g. [4, 10, 15] and references therein). The only point

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[5]It should not be viewed that the operator in eq. (2.7) is obtained by setting the coefficients of the two operators in eq. (5.1) equal by hand. Because of the "chiral" Z_2 symmetry, $(H_A|H_B) \leftrightarrow (H_A|H_B)$ and $(H'_A|H'_B) \leftrightarrow (H'_A|-H'_B)$, these coefficients are necessarily equal, $\lambda_1 = \lambda_2$, so that the operator of eq. (2.7) is really a single operator.

worth mentioning is that our models provide a natural way to lift the mirror photon mass because $SU(2)_B \times U(1)_B$ is completely broken in the vacuum. This relaxes many of the cosmological constraints, related to the excess in radiation energy density coming from the mirror sector and to the production of proto-Galaxies from the dark matter mirror baryons.

Regarding collider physics, our models are quite distinct. At low energies the "visible" particle content of both models is that of a general two Higgs doublet standard model. At higher energies additional singlet scalar particles are present. In the model with the extended top sector there are new fermions charged under the standard model gauge group with masses of $\approx (1 \sim 2)$ TeV, but these particles are absent in the minimal model of section 2.

In general we are interested in parameter regions where no severe fine-tuning is required. In these regions the lightest Higgs boson is relatively heavy, with masses of $O(150 \sim 250$ GeV), allowing for an easy detection through WW decays. However, the detection of all five Higgs bosons at the LHC is, in general, non-trivial (see e.g. [16] for recent reviews). Thus, without detecting the other mirror particles or singlet fields, our model would look simply like a two Higgs doublet standard model or perhaps even just the standard model.

It is important to consider whether one can have additional signals at the LHC that allow to distinguish the models from a simple two Higgs doublet standard model. In the model of section 4, there are colored fermions of masses $\approx (1 \sim 2)$ TeV, which can be found easily at the LHC. In the model of section 2, however, the detection of new physics beyond the two Higgs doublet standard model will, at best, be a difficult task, because all the new particles are singlet under the standard model gauge group.

The simplest possibility would be to look for invisible decays of the Higgs bosons into mirror fermions [17]. The relevant vertices, however, arise only from the mixing between the neutral scalars of the two sectors, so that they are all suppressed by powers of v/f or v/f'. The most important decay channels would be to the mirror bottom quark, which is the heaviest mirror particle available below the Higgs boson masses. The branching ratios to these invisible decay modes, however, are still too small to be observed at the LHC, which requires the product of the Higgs production cross section normalized to the standard model one and the branching fraction into the invisible channel to be about 0.1 or larger [18].

The situation is similar in pseudoscalar and charged Higgs boson decays. For the pseudoscalar case, the vector boson fusion channel is not available because it does not couple to the gauge bosons at tree level. This makes it almost impossible to detect the invisible width because of the standard model background. For the charged Higgs boson case, one might try to tag the associated production of a visible charged particle and invisible fields. The decay modes of the charged Higgs boson into something visible and mirror particles, however, proceed only through higher dimension operators and are highly suppressed. The other possibility would be to look at the cascade of a charged Higgs into mirror particles through a neutral Higgs, for example as $H^+ \to Wh^0 \to l + $ missing energy, where H^+ is produced in the standard way through $gb \to tH^+$ [19]. This requires, however,

to fully reconstruct the top quark hadronically, and a large standard model background from $gg \to \bar{t}t$ with one top quark decaying semileptonically makes the observation of these "semi-invisible" decay impractical [20].

A possibility of distinguishing our model from a simple two Higgs doublet standard model may come from the decay of the radial excitation modes of Φ and Φ', which have the masses $\sqrt{2\eta}f$ and $\sqrt{2\eta'}f'$. Since the light Higgs boson has mixings with these modes of $O(v/f)$, a heavy radial mode can be produced instead of a Higgs boson [21] with a production cross section a factor of $\simeq v^2/f^2$ below that of the Higgs boson. The radial modes have couplings of $O(f)$ to a pair of the light Higgs bosons. Thus, after being produced on-shell, the radial mode can decay into two light Higgs bosons, which then decay into standard model particles. This may be the dominant decay of the radial modes; the only competing ones would be decays into a mirror top or gauge boson pair, which could be kinematically forbidden because all of these particles have masses of $O(f)$. We expect that the masses of the CP even Higgs bosons are above the ZZ threshold, so one can look at the light Higgs bosons, which are produced by the decay of a radial mode and decay into ZZ and WW pairs. A rough estimate, however, shows that the channel in which one Z and one W decay leptonically does not have a large event rate. Thus, even though the standard model background is small, it is difficult to observe this channel (unless the mass of the radial mode is somewhat unexpectedly small). One may also look for (one of) the Higgs bosons decaying either into b or τ pairs, which increases the event rate. This, however, also increases the standard model background, so that a more detailed analysis is needed to see if this mode is useful. We also note that if η and η' are of order unity or somewhat larger, the radial modes become (much) heavier than a TeV, and the detection of these modes at the LHC becomes almost impossible. Following the discussion after eq. (3.23), this may be the case for smaller fine-tuning.

At a linear collider, invisible decays of the Higgs bosons may be accessible. The branching ratios, however, are not so large $\approx 10^{-3}$ for the associated production with a Z boson, $ZH \to ll + $ missing energy, so that it is not clear if this can be detected. Another possible channel is to produce a pseudoscalar associated with a neutral Higgs boson, and look for an invisible decay of the pseudoscalar Higgs boson, which has a branching ratio of order $(v/f)^2$. The precise study of the masses and couplings of the Higgs bosons may also be used to discriminate between our model and the simple two Higgs doublet standard model.

Finally, precision electroweak constraints are easily satisfied by construction [4]. In the model of section 2 all particles beyond those in the two Higgs doublet standard model are singlet under the standard model gauge group, and the effects on the precision electroweak observables are small. The contributions from heavy radial modes, for example, come only through mixings with light Higgs bosons, which induce an additional logarithmic contribution which effectively appears as arising from two very heavy Higgs bosons but with the coefficients suppressed by factors of $\mathcal{O}(v^2/f^2)$. The contributions from vector-like fermions in the model of section 4 are also small, as they have $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant masses of order $(1\sim2)$ TeV. Dangerous operators induced by ultraviolet physics are either suppressed by assumed symmetries or, if it is not possible, as in the case of the operator for the S parameter, are suppressed by our rather high cutoff scale of $(5\sim8)$ TeV.

7. Conclusions

In this paper we have constructed a theory in which radiative corrections to the quadratic part of the potential are constrained to be symmetric under a global $U(4) \times U(4)'$ symmetry due to a discrete Z_2 symmetry, while the quartic part does not possess this symmetry at all. The theory is weakly coupled at the cutoff scale Λ, and has a simple structure where the two Higgs doublet standard model is simply "twinned" due to the Z_2 symmetry. The two Higgs doublets have a quartic coupling at tree level, while their squared masses are generated only at order $f^2/16\pi^2$, where f is an order parameter for the $U(4) \times U(4)'$ breaking, which is supposed to be a factor of 4π smaller than Λ. This setup, thus, potentially allows us to have a large hierarchy between the electroweak VEV, v, and the cutoff scale.

We have carefully studied fine-tuning in this theory and found that we do not obtain a hierarchy as large as what may naively be expected, $\Lambda \simeq 4\pi f \simeq 16\pi^2 v$. The theory, however, still allows a reduction of fine-tuning by a factor of ≈ 4 compared with the standard model, even in the minimal version, which allows us to push up the cutoff scale to about 5 TeV without significant fine-tuning ($\Delta^{-1} \approx 14\%$). This is almost enough to solve the little hierarchy problem, implied by the mismatch between the stability of the electroweak scale and the constraints from experiments. With the $U(4)$-extended top quark sector, we can further reduce fine-tuning to the level of 30% for $\Lambda \approx 5$ TeV, or if we allows a mild tuning of order 10% the cutoff scale can be as high as $\Lambda \approx 8$ TeV. In general, the theory prefers a heavy Higgs boson, $m_{h^0} \simeq (150 \sim 250)$ GeV, and small values for the ratio of the VEVs for the two Higgs fields, $\tan\beta \simeq (1 \sim 2)$.

Our theory provides an example of a potentially embarrassing situation at the LHC. While the theory is not significantly fine-tuned, the LHC may just see the two Higgs doublet standard model, and may not find any new physics responsible for cutting off the divergences of the Higgs mass-squared parameter. This occurs in the model without extra vector-like fermions. All quadratic divergences in the Higgs mass-squared parameter due to standard model loops are canceled by fields that are singlet under the standard model gauge group. The deviations from the simple two Higgs doublet model due to these singlet fields can be very small at the LHC. We have discussed several possible processes that may be able to discriminate our model from the two Higgs doublet model at the LHC and at a linear collider. It will be interesting to study these processes in more detail.

Possible physics above the cutoff scale Λ is unknown. We have discussed the possibility of extending the theory to the strongly coupled regime at Λ. It would be interesting to pursue possible ultraviolet physics that reduces to our theory below the scale of $\Lambda \approx (5 \sim 8)$ TeV.

Note added. While completing this paper, we received ref. [22], which also addresses the little hierarchy problem in the context of the two Higgs doublet standard model.

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